

**3 SEM TDC MTMH (CBCS) C 7**

**2 0 2 1**

( Held in January/February, 2022 )

**MATHEMATICS**

( Core )

Paper : C-7

( PDE and Systems of ODE )

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) Write the degree of the equation

$$x \left( \frac{\partial^2 z}{\partial x^2} \right) + \left( \frac{\partial z}{\partial y} \right)^2 = \frac{\partial z}{\partial x}$$

1

- (b) Write Lagrange's subsidiary equation of

$$xzp + yzq = xy$$

1

- (c) Write the general solution of  $pq = k$ .

1

- (d) Solve :

5

$$(y - zx)p + (x + yz)q = x^2 + y^2$$

Or

Find the integral surface of  $x^2 p + y^2 q + z^2 = 0$ , which passes through the hyperbola  $xy = x + y, z = 1$ .

- (e) Show that the equations  $xp - yq = x$  and  $x^2 p + q = xz$  are compatible.

5



( 2 )

2. (a) Write Charpit's auxiliary equations for  $q = 3p^2$ . 2
- (b) Find complete integral of any one of the following : 4
- (i)  $q = (z + px)^2$
- (ii)  $q + px = p^2$
- (iii)  $z^2 = pqxy$
- (c) Find a complete integral of

$$p_1^3 + p_2^2 + p_3 = 1 \quad 6$$

Or

Solve the boundary value problem  $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$  with  $u(0, y) = 8e^{-3y}$  by the method of separation of variables.

3. (a) Write the condition when the equation  $Rs + Ss + Tt + f(x, y, z, p, q) = 0$  is hyperbolic. 1

- (b) Determine the nature of the equation  $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$  2

- (c) Show that  $u = f(x+y) + g(y-x)$  satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

where  $f$  and  $g$  are functions. 2

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( Continued )

( 3 )

- (d) Reduce the equation  $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$  to canonical form. 7

Or

Derive the one-dimensional heat equation.

4. (a) Write the general form of two-dimensional heat equation. 1
- (b) Write one assumption on vibrating string problem. 1
- (c) Solve

$$\frac{\partial^2 u}{\partial x^2} = k^2 \left( \frac{\partial u}{\partial t} \right)$$

when  $u(0, t) = u(l, t) = 0$ ,  $u(x, 0) = \sin \frac{\pi x}{l}$ . 6

Or

Solve the two-dimensional heat equation by the method of separation of variables.

5. (a) Write the equation  $3 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} - x = t^2$  in normal form. 1
- (b) Let  $L \equiv D^2 + 2$ ,  $f(t) = e^{2t}$ , where  $D \equiv \frac{d}{dt}$ . Find  $Lf(t)$ . 2

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( Turn Over )



- (c) Transform the linear differential equation

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 2x = t^2$$

into a system of first-order differential equation.

2

- (d) Describe Euler's method.

4

Or

Find the characteristic roots of the equation associated in the solution of

$$\frac{dx}{dt} = 3x + y, \quad \frac{dy}{dt} = 4x + 3y$$

- (e) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} - x - 3y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + x = e^{3t}$$

6

Or

Find  $y(0.1)$ ,  $y(0.2)$  in the solution of  $\frac{dy}{dx} = x + y$ ,  $y(0) = 1$ , by using Runge-Kutta method.

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